Synopsis for Ph.D. Programme

Title of the Research:
Study of some water wave scattering by obstacles

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The theoretical study of problems of scattering of surface water waves by barriers involves several important and interesting concepts of mathematics and physics of flow of ideal fluids. The first rigorous mathematical analysis was developed and applied in water wave scattering problems by Ursell in 1947[1]. Since then a significant number of developments have taken place in this interesting area of Applied Mathematics.

In the mathematical study of water wave problems, two type of theories are employed, one is the linearized theory of water waves and the other is non-linear shallow water theory. If the wave-length is assumed to be much less than the depth of water, then the effect of disturbance diminishes gradually as one moves downwards. If it is assumed that the wave amplitude is small compared to the wavelength, the corresponding theory of water waves is called linearized theory of water waves[2].

A different kind of approximation from the linear theory of waves of small amplitude results when one assumes that the depth of water is sufficiently small compared to some significant length (such as wave length). Then it is not necessary to assume that the derivation of the upper surface of water and its slope are small and the resulting theory is no longer a linear theory. Such a theory leads to a effective approximation to various natural phenomena such as the tides in the oceans, breaking of waves on shallow beaches etc.

When a train of surface water waves traveling from a large distance is incident on an obstacle submerged or partially immersed in water, some part of the wave reflected back by the obstacle and some part is transmitted over or below it. This type of problem is known as scattering problem. The reflected and transmitted wave fields involve two constant factors known as reflection and transmission co-efficient respectively. It is supposed that the incident wave field is completely known. But the resulting wave field after interaction is unknown. Determination of the unknown scattered wave field together with the unknown reflection and transmission coefficients constitutes a scattering problem. Water wave scattering problems have practical applications such as modeling of break waters, designing ships, offshore structures etc.

Again if some part of sea surface is covered by some materials such as broken ice or a rigid plate or an elastic plate, where the material meets the surface. Here also the reflection and transmission phenomenon of the incident wave occurs.

Let (x,y) be the rectangular Cartesian co-ordinate where y-axis is chosen vertically downwards. Then the basic equations for the linearized theory of water waves are-

- $\nabla^2\Phi = 0$ in the fluid region.
- $\frac{\partial^2\Phi}{\partial t^2} = g\frac{\partial\Phi}{\partial y}$ on the free surface $y = 0$.
- $\nabla\Phi \longrightarrow 0$ as $y \longrightarrow 0$ (in case of deep water).

or $\frac{\partial\Phi}{\partial y} = 0$ on $y = h$ (in the case of uniform finite depth).
where $\Phi(x, y, t)$ is the velocity potential and g is the gravitational constant.

For the scattering problem let a train of time harmonic progressive waves of angular frequency σ represented by $\phi^{inc}(x, y)e^{i\sigma t}$ traveling from a large distance at negative infinity be incident on a stationary body present in water. Let the resulting motion be described by a velocity potential $\mathrm{Re}\phi(x, y)e^{i\sigma t}$.
Then ϕ satisfies:

- $\nabla^2\phi = 0$ in the fluid region.
- $K\phi + \frac{\partial\phi}{\partial y} = 0$ on $y = 0$.
- $\frac{\partial\phi}{\partial n} = 0$ on the wetted part of the body (n is outward drawn normal to the body).
- $\nabla\phi \longrightarrow 0$ as $y \longrightarrow 0$ (in case of deep water).

or $\frac{\partial\phi}{\partial y} = 0$ on $y = h$ (in the case of uniform finite depth).
and $\phi(x, y)$ satisfies the condition at infinity as described by,

$$\phi(x,y) \longrightarrow \begin{cases} \phi^{inc}(x,y) + R\phi^{inc}(-x,y) & \text{as } x \longrightarrow -\infty \\ T\phi^{inc}(x,y) & \text{as } x \longrightarrow \infty \end{cases}$$

where R and T are reflection and transmission coefficients respectively. The incident fields are given by

$$\phi^{inc}(x,y) = \begin{cases} e^{-Ky+iKx} & \text{for deep water} \\ \frac{Coshk_0(h-y)}{Coshk_0h}e^{ik_0x} & \text{for water of uniform finite depth h} \end{cases}$$

Cloaking is a phenomenon generally, related to electromagnetism and optics. There has been crossover of ideas from optics and electromagnetism into the acoustic and elasticity research communities. Much less connection has been realized in water wave community as water waves are quite different to electromagnetism, elasticity or acoustic waves. The application of metameterials in water waves are harder to imagine. On the other hand wave phenomena are easily visualized on the surface of water and this makes it an attractive medium in which to develop ideas of cloaking from other areas of physics.

In 2008, Farhat et al[3]. published their work titled "On Broad-band cylindrical acoustic clock for linear surface waves in fluid" on cloaking of water waves. After that Griffith[4], Porter[5] developed mathematical models related to cloaking of water waves.

My main aim is to study the mathematical problems involving scattering of water waves and solve those problems using effective and efficient numerical methods with help of mathematical softwares like Mathematica, Maple, Matlab etc.

References

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